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## Clinical problem

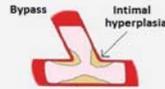
Coronary artery disease is caused by the buildup of atherosclerotic plaques in the coronary vessel walls

Reduction of oxygen supply to the heart, leading to **ischemia, ventricular fibrillation and heart failure**



**Surgical treatment:** coronary artery bypass graft

**Possible complication:** bypasses in patients with non-severe stenosis (<70%) may fail due to **anastomotic restenosis** [1]



### Objective

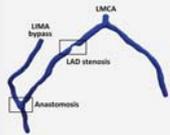
We look for a possible **relationship between stenosis degree and risk of graft failure** in terms of WSS-based hemodynamic indices potentially involved in **intimal hyperplasia** development

## Materials and Methods

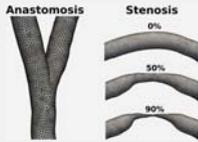
### From CT scans to computational domains

With software VMTK (www.vmtk.org):

- Level-set segmentation to reconstruct the *patient-specific geometries*
- Ad-hoc scripts to *virtually change the degree of coronary stenosis*
- Surfaces turned into *volumetric meshes of tetrahedra*



LMCA = Left main coronary artery  
 LIMA = Left interior mammary artery  
 LAD = Left anterior descending



### Extension of Murray's law

In order to prescribe *realistic boundary conditions* in our numerical simulations, we need to estimate the *flow division* at bifurcations

**Murray's law** [2]:  $\frac{Q_d}{Q_p} = \left(\frac{r_d}{r_p}\right)^3$ , d = daughter, p = parent

However, **Murray's law is valid only for healthy vessels and Newtonian fluids**

- We propose an **extended Murray's law** to account for:
- presence of a **stenosis** in the daughter vessel
  - non-Newtonian rheology** of blood

**Extended Murray's law:**  $\frac{Q_d}{Q_p} = \frac{Q_d^{sten}}{Q_p} = \frac{Q_d^{sten} (r_d, \alpha, \mu_{1,d}, \mu_{2,d}, \mu_{3,d}, l_{1,d}, l_{2,d}, l_{3,d})}{\sqrt{\frac{\mu_p^2 r_p^3}{10 \mu_p r_p}}}$   
 for a suitable function  $Q_d^{sten}$ \*

\* see [3] for more details about the expression of  $Q_d^{sten}$  and its derivation

If the daughter vessel is not stenotic ( $\alpha = 1$ )  $\rightarrow \frac{Q_d}{Q_p} = \sqrt{\frac{\mu_p}{\mu_d}} \left(\frac{r_d}{r_p}\right)^3$

N.B. In case of Newtonian flow ( $\mu_p = \mu_d$ ), the original Murray's law is recovered

### Governing equations and boundary conditions

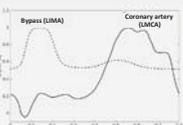
Discretized-in-time Navier-Stokes equations:  $\begin{cases} \rho_t \mathbf{u}^n - \mathbf{u}^{n-1} + \rho_t \mathbf{u}^{n-1} \cdot \nabla \mathbf{u}^n - \nabla \cdot [\mu(\mathbf{u}^{n-1}) (\nabla \mathbf{u}^n + (\nabla \mathbf{u}^n)^T)] + \nabla p^n = 0 \\ \nabla \cdot \mathbf{u}^n = 0 \end{cases}$

- Shear-thinning behavior of blood described by **Carreau-Yasuda** model [4]:

$$\mu(\mathbf{x}, t) = \mu_\infty + (\mu_0 - \mu_\infty) (1 + (\lambda \dot{\gamma}(\mathbf{x}, t))^a)^{\frac{n-1}{a}}$$

- Semi-implicit treatment of **convective** and **diffusive** terms
- Equations numerically approximated and solved with the **Finite Element library LifeV\*\*** (www.lifev.org)

\*\* developed at MOX - Politecnico di Milano, INRIA - Paris, CMCS - EPF de Lausanne, Emory University - Atlanta



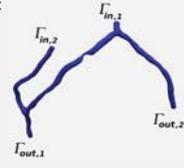
Patient-specific velocity/pressure data unavailable

We chose from literature **representative flow rate waveforms** for the coronary artery and bypass inlets

For the LIMA bypass, the amplitude of the signal is set in accordance to the stenosis degree, in order to guarantee that the **flow rate perfusing the myocardium** (calibrated by means of a **healthy simulation**) **remains constant**, as observed in the clinical practice [1] (see [3] for more details)

At each time  $t^n$ , we solve the NS equations with the following BCs:

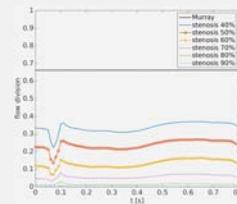
- Flow rate condition at  $\Gamma_{in,1}$ :  $\int_{\Gamma_{in,1}} \mathbf{u}^n \cdot \mathbf{n} d\gamma = Q_{LMCA}(t^n)$
- Flow rate condition at  $\Gamma_{in,2}$ :  $\int_{\Gamma_{in,2}} \mathbf{u}^n \cdot \mathbf{n} d\gamma = Q_{LIMA}^X(t^n)$
- Flow rate condition at  $\Gamma_{out,1}$ :  $\int_{\Gamma_{out,1}} \mathbf{u}^n \cdot \mathbf{n} d\gamma = Q_{LIMA}^X(t^n) + Q_{LAD}^X(t^n)$
- Zero-traction condition at  $\Gamma_{out,2}$ :  $-\rho^n \mathbf{n} + \mu(\mathbf{u}^{n-1}) (\nabla \mathbf{u}^n + (\nabla \mathbf{u}^n)^T) \mathbf{n} = 0$



Flow rate conditions are imposed with a **Lagrange multipliers** approach [5]

## Results

### Flow divisions with different stenosis degree



- The **original Murray's law overestimates the flow division** with respect to our extension
- The proposed **extended Murray's law** detects different degrees of stenosis, providing **decreasing flow divisions** for increasing stenosis degree

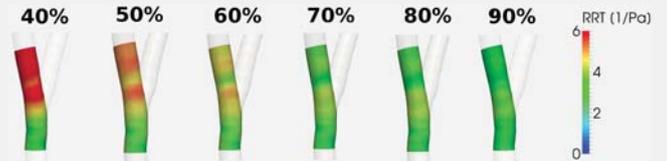
### Risk of anastomotic restenosis

- The **Relative Residence Time (RRT)** is related to intimal hyperplasia development

$$RRT(\mathbf{x}) = \frac{1}{(1 - 2OSI(\mathbf{x}))TAWSS(\mathbf{x})}$$

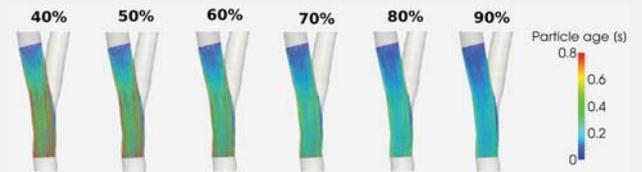
$$OSI(\mathbf{x}) = \frac{1}{2} \left( 1 - \frac{\int_0^T \tau_w(t, \mathbf{x}) dt}{\int_0^T \|\tau_w(t, \mathbf{x})\| dt} \right) \quad \tau_w = \text{Wall Shear Stress}$$

$$TAWSS(\mathbf{x}) = \frac{1}{T} \int_0^T \|\tau_w(t, \mathbf{x})\| dt$$



The **lower the degree of stenosis**, the higher the RRT and thus the **higher the risk of anastomotic restenosis** (graft failure)

- Particle tracking techniques** allow to follow particles and to quantify the **residence time** spent by each particle inside a region of interest



**Stagnation regions** are more likely to occur in cases with non-severe stenosis (longer particle ages), possibly **enhancing the risk of restenosis**

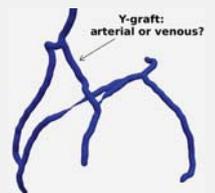
## Conclusions and future perspectives

- The **extended Murray's law** may be a **suitable alternative** to the **prescription of realistic BCs** when clinical data are not available
- Lower LAD stenosis degrees** result in higher RRT values and longer residence times, possibly enhancing the **risk of LIMA graft restenosis**

### Future developments:

- Validation of the extended Murray's law
- Fluid-structure interaction (FSI)** modeling of the coronary tree with multiple bypasses (Y-grafts)

Analyze the influence of the type of bypass (arterial or venous) on the risk of anastomotic restenosis due to **compliance mismatch**



## References

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 [2] C. D. Murray, "The physiological principle of minimum work", Proc. Natl. Acad. Sci., vol. 12, pp. 207-214, 1926  
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 [4] S. S. Shiveshi and W. E. Collins, "The rheology of blood flow in a branched arterial system", Appl. Rheo., vol. 15, pp. 398-405, 2005  
 [5] A. Veneziani and C. Vergara, "Flow rate defective boundary conditions in haemodynamics simulation", Int. J. Numer. Meth. Fl., vol. 47, pp. 803-816, 2005